

# **Tuesday 21 June 2016 – Morning**

# **A2 GCE MATHEMATICS**

4735/01 Probability & Statistics 4

#### **QUESTION PAPER**

Candidates answer on the Printed Answer Book.

#### **OCR** supplied materials:

- Printed Answer Book 4735/01
- List of Formulae (MF1)

#### Other materials required:

Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

# **INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer all the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do not write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

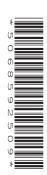
## **INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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# Answer all the questions.

1 Ten archers shot at targets with two types of bow. Their scores out of 100 are shown in the table.

Archer	A	В	C	D	E	F	G	Н	I	J
Bow type P	95	97	92	85	87	92	90	89	98	77
Bow type Q	91	91	88	90	80	88	93	85	94	84

- (i) Use the sign test, at the 5% level of significance, to test the hypothesis that bow type P is better than bow type Q.
- (ii) Why would a Wilcoxon signed rank test, if valid, be a better test than the sign test? [1]
- Low density lipoprotein (LDL) cholesterol is known as 'bad' cholesterol.
  15 randomly chosen patients, each with an LDL level of 190 mg per decilitre of blood, were given one of two treatments, chosen at random. After twelve weeks their LDL levels, in mg per decilitre, were as follows.

Use a Wilcoxon rank sum test, at the 5% level of significance, to test whether the LDL levels of patients given treatment *B* are lower than the LDL levels of patients given treatment *A*. [8]

3 The table shows the joint probability distribution of two random variables *X* and *Y*.

		Y				
		0	1	2		
	0	0.07	0.07	0.16		
X	1	0.06	0.09	0.15		
	2	0.07	0.14	0.19		

(i) Find 
$$Cov(X, Y)$$
. [5]

(ii) Are *X* and *Y* independent? Give a reason for your answer.

[2]

(iii) Find 
$$P(X = 1 | XY = 2)$$
.

4 The continuous random variable Y has a uniform (rectangular) distribution on [a, b], where a and b are constants.

(i) Show that the moment generating function 
$$M_Y(t)$$
 of  $Y$  is  $\frac{(e^{bt} - e^{at})}{t(b-a)}$ . [2]

(ii) Use the series expansion of 
$$e^x$$
 to show that the mean and variance of  $Y$  are  $\frac{1}{2}(a+b)$  and  $\frac{1}{12}(b-a)^2$ , respectively. [7]

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- 5 Events A and B are such that P(A) = 0.5, P(B) = 0.6 and P(A|B') = 0.75.
  - (i) Find  $P(A \cap B)$  and  $P(A \cup B)$ .

- (ii) Determine, giving a reason in each case,
  - (a) whether A and B are mutually exclusive,
  - (b) whether A and B are independent.

[2]

[1]

- (iii) A further event C is such that  $P(A \cup B \cup C) = 1$  and  $P(A \cap B \cap C) = 0.05$ . It is also given that  $P(A \cap B' \cap C) = P(A' \cap B \cap C) = x$  and  $P(A \cap B' \cap C') = 2x$ . Find P(C). [3]
- Andrew has five coins. Three of them are unbiased. The other two are biased such that the probability of 6 obtaining a head when one of them is tossed is  $\frac{3}{5}$ .

Andrew tosses all five coins. It is given that the probability generating function of X, the number of heads obtained on the unbiased coins, is  $G_X(t)$ , where

$$G_X(t) = \frac{1}{8} + \frac{3}{8}t + \frac{3}{8}t^2 + \frac{1}{8}t^3$$
.

- (i) Find  $G_Y(t)$ , the probability generating function of Y, the number of heads on the biased coins. [3]
- (ii) The random variable Z is the total number of heads obtained when Andrew tosses all five coins. Find the probability generating function of Z, giving your answer as a polynomial. [3]
- (iii) Find E(Z) and Var(Z). [6]
- (iv) Write down the value of P(Z = 3).
- 7 A continuous random variable Y has cumulative distribution function

$$F(y) = \begin{cases} 0 & y < a \\ 1 - \frac{a^5}{y^5} & y \ge a \end{cases}$$

where a is a parameter.

Two independent observations of Y are denoted by  $Y_1$  and  $Y_2$ . The smaller of them is denoted by S.

- (i) Show that  $P(S > s) = \frac{a^{10}}{s^{10}}$  and hence find the probability density function of S. [5]
- (ii) Show that S is not an unbiased estimator of a, and construct an unbiased estimator of a,  $T_1$  based on S. [4]
- (iii) Construct another unbiased estimator of a,  $T_2$ , of the form  $k(Y_1 + Y_2)$ , where k is a constant to be found.
- (iv) Without further calculation, explain how you would decide which of  $T_1$  and  $T_2$  is the more efficient estimator. [1]

## **END OF QUESTION PAPER**

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